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GRADE 9

The following math problems are representative of what 9th-grade students in India are expected to master to be promoted to grade 10.

As a 9th-grader here in America, we hope you will try out some or all of these problems, and perhaps learn some geography and history about India too!

If you are stuck or are not satisfied with your solutions, you can always ask for help from your peers and friends through BeyondGPA's q&a service.

You should also monitor questions posted by your peers and help if you can. After all, it is students who can best help other students.

India uses the metric (mks) system.
Some units & conversion factors:

1 US Dollar = 40 Indian Rupees (Rs.)
(The rate can vary but use this for consistency)

1 mile (mi) = 1.6 kilometers (km)
1 inch = 2.54 cm = 25.4 mm
1 meter (m) = 100 centimeter (cm) = 1000 millimeter (mm)
1 kilogram (kg) = 1000 gram (g)

1 kilogram = 2.2 pound
1 pound = .45 kilogram
1 gallon = 4.4 liters
(Liter is the basic unit of volume in the metric system.
It is spelled Litre in India & Europe.)
1 liter (l) = 1000 milliliter (ml)

1. Evaluate: a) $(2^{65} \times 2^{47}) - (2^{33} \times 2^{79})$ b) $(2/3)^2 \times (2/5)^{-3} \times (3/5)^2$

2. Simplify:
   i) $(625)^{-1/4}$
   ii) $(243/32)^{-4/5}$
   iii) $(81/16)^{-3/4} \times [(25/9)^{-3/2} / (5/2)^{-3}]$

3. Show that
   a) $1/(1 + x^{b-a} + x^{c-a}) + 1/(1 + x^{a-b} + x^{c-b}) + 1/(1 + x^{b-c} + x^{a-c}) = 1$

   b) $(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2/(x^a x^b x^c)^4 = 1$

4. Find the products
   a) $(x - 1/x)(x + 1/x)(x^2 + 1/x^2)(x^4 + 1/x^4)$
   b) $(7x - 5y)(49x^2 + 35xy + 25y^2)$

5. a) If $4x^2 + y^2 = 40$ and $xy = 6$, find the value of $2x + y$.
   b) if $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$

6. a) If $x + y + z = 0$, prove that $x^3 + y^3 + z^3 = 3xyz$
   b) Evaluate $40^3 – 65^3 + 25^3$

7. Factorize the following expressions:
   i) $x^4 + 4$
   ii) $x^4 + 2x^2 + 3$
   iii) $1 – 2pq – (p^2 + q^2)$

8. What are the possible dimensions of a cube whose volume is $3a^2 – 12a$? List as many combinations as you can.

9. Factorize the following: a) $p^6 – q^6$ b) $p^6 + q^6$ c) $p^7 + pq^6$ d) $p^{12} – q^{12}$

10. For the polynomial $f(x) = 2x^3 - 13x^2 + 17x + 12$, find f(0), f(-1/2), f(2) and f(3). Which is a root of the polynomial?

11. If $y = 4/3$ is a root of the polynomial $f(y) = 6y^3 - 11y^2 + ky – 20$, find the value of k.

12. If the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$ has two roots, $x = 2$ and $x = 0$, find the values of a and b.

13. The remainder theorem states that the remainder obtained when f(x) is divides by x-a is equal to f(a), that is, the value of f(x) when x = a. Thus, the remainder when $f(x) = x^3 + x^2 + 2x + 3$ when divided by (x+2) is obtained from f(-2) = -5. Prove this by dividing the given polynomial f(x) by (x+2).

14. Using the remainder theorem, find the remainder when
   a) $f(x) = 4x^3 - 12x^2 + 14x - 3$ is divided by $g(x) = x – ½$

   b) $f(x) = x^3 – 6x^2 + 2x - 4$ is divided by $g(x) = 3x – 1$

15. If the two polynomials $ax^3 + 3x^2 - 13$ and $2x^3 – 5x + a$ are divided by (x+2), the remainder in each case is the same. Find the value of a.

16. If the polynomial $f(y) = y^4 - 2y^3 + 3y^2 – ay + b$ is such that when it is divided by (y-1) and (y+1), the remainders are 5 and 19 respectively. Find the remainder when f(y) is divided by (y-2).

17. a) Show that (x + 1) and (2x – 3) are factors of $2x^3 – 9x^2 + x + 12$

   b) Find the value of a if (x-a) is a factor of $x^3 – a^2x + x + 2$

18. Draw the graphs of the following linear equations:
   a) $2x + 4 = x + 7$
   b) $2y + 4 = 12$

19. Draw the graphs of the equations $3x - 2y = 4$ and $x + y = 3$ in the same graph and find the coordinates of the points where the two lines intersect.

20. a) If the angles of a triangle are in the ratio of 3:4:5, what are its three angles?

    b) x, y and z are the three angles of a triangle. If $x - y = 15^0$ and $y - z = 30^0$, find the three angles.

    c) If one angle of a triangle is equal to the sum of the other two angles, show that it is a right triangle.

21. a) Prove that if the altitude from one vertex of a triangle bisects the opposite side, the triangle is isosceles

    b) Show that the difference of any two sides of a triangle is less than the third side.

22. a) Two opposite angles of a parallelogram are $(3x-2)^0$ and $(40+x)^0$. Find the measure of each angle of the parallelogram.

    b) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, it is a square.

23. a) The radius of a circle is 13 cm and the length of one of its chords is 10 cm. Find the distance of the chord from its center.

    b) AB and CD are two parallel chords of a circle whose center is O and radius is 10 cm. If AB = 16 cm and CD = 12 cm, find the distance between AB and CD i) if they are on the same side of the center O ii) if they are on the opposite side of the center O.

24. a) Find the area of a triangle, two sides of which are 8 cm and 11 cm and its perimeter is 32 cm.

    b) The perimeter of a triangular field is 450 m and its sides are in the ratio of 5:12:13. Find its area.

25. a) In America, a five-sided traffic sign (a triangle on a rectangle) showing two students means that you are near a school. In India, 'SCHOOL AHEAD' traffic sign is often an equilateral triangle. If an Indian school traffic sign has a perimeter of 180 cm, find its area. Find the area of the American school sign if it has the same triangular dimensions but the sides of the rectangle measures 80 cm each.

    b) A triangle and a parallelogram have the same base and the same area. If the sides of the triangles are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the 28 cm base, find the height of the parallelogram.

26. Farmer Anil has a plot of land in the shape of a rhombus. He wants his two sons to work on the land to produce rice and vegetables for the family. He divides the land into two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area will be available to each of

the sons to cultivate?

27. a) The four walls of a room in a residential building in Kolkata (formerly Calcutta) have these measurements: length = 6 m, width = 5 m, height = 4 m. Find the area of the four walls. Ignoring any openings, if the walls are painted blue at the rate of Rs. 35 per square meter, find the cost pf painting the walls.

    b) A swimming pool in a Mumbai (formerly Bombay) hotel is 20 m in length, 15 m in width and 4 m in depth. Find the cost of cementing the walls and the floor of the pool at the rate of Rs. 75 per square meter.

28. A plot of land in a construction zone in Gurgaon has a dimension of 240 m x 180 m. A drain 10 m wide is dug all around it on the outside and the earth dug out is spread evenly over the plot, raising its surface level by 25 cm. Find the depth of the drain.

29. A rectangular water reservoir in Jaipur is 10.8 m by 3.75 m at the base. Water flows into it at the rate of 18 m per second through a pipe with a cross-section of 7.5 cm x 4.5 cm. Find the height through which the water will rise in the reservoir in 1 hour.

30. Water flows into a tank 150 m x 100 m at the base at the speed of 15 km per hour through a pipe whose cross-section is 2 dm x 1.5 dm (decimeter; 10 dm = 1 m). After how long will the water be 3 meters deep?

31. The diameter of a roller 120 cm long is 84 cm. If it takes 900 complete revolutions to level a children's playground, find the cost of leveling at the cost of Rupees 5 per square meter.

32. a) Find the volume of a sphere whose surface area is 176 square cm.

    b) The volumes of two spheres are in the ratio of 125:64. Find the difference of their surface areas if the sum of their radii is 9 cm.

33. The highest temperature in degree Celsius and relative humidity in percent for Delhi in the month of August for a certain year are as follows (the high relative humidity explains why it gets so sweaty in summer in the Indian subcontinent):

    Highest temperatures in degrees Celsius for each day in August: 32.5, 30.5, 33.8, 31.0, 28.6, 33.9, 33.3, 32.4, 30.4, 32.6, 34.7, 34.9, 31.9, 35.2, 35.3, 35.5, 36.4, 36.9, 37.0, 34.4, 32.5, 31.4, 34.4, 35.6, 37.3, 37.5, 36.9, 37.0, 36.3, 36.9, 36.7

    Relative humidity in percent for each day in August: 90, 97, 92, 95, 93, 95, 93, 85, 83, 85, 83, 77, 83, 77, 74, 60, 71, 65, 74, 80, 87, 82, 81, 76, 61, 63, 58, 58, 56, 57, 54

    Construct a frequency table for each. (Use range, which is the difference between the highest and lowest values, and an appropriate interval size)

34. Factory workers in India often earn less than a dollar a day. For the following data of daily wages received by 30 workers in a factory in Assam, construct a frequency distribution with an interval of Rs. 2.

    24, 26, 26, 24, 32, 23, 25, 34, 22, 33, 24, 30, 27, 31, 32, 28, 28, 29, 30, 27, 26, 25, 21, 22, 31, 30, 27, 28, 29, 33.

35. From the cumulative frequency distribution table below, construct a regular frequency distribution table.

Points	Below 25	Below 40	Below 55	Below 70	Below 85	Below 100
Number of students	1	8	24	50	82	120

36. The mean, or average, of 40 observations is 180. Later it is found that the value of two of the observations, 185 and 245, were wrongly copied as 125 and 225. Find the correct mean.

37. a) Find the median of the following values:
    a) 47, 41, 53, 52, 35, 56, 49, 55, 52

    b) 45, 54, 51, 43, 42, 46, 55, 48, 40, 52, 51, 48

38. a) The number of different-sized garments produced by garment factories is often based on their modal distributions. From the following distribution of full-sleeve shirts in a given survey, find the modal shirt size. (Shirt sizes in India are measured in cm. In the U.S., it is measured in inches.)

Shirt size (cm)	38	39	40	41	42	43
Number of persons wearing it	26	42	22	17	9	6

    b) List some of the differences between mean, median and mode (also known as measures of central tendency) and where you would use one rather than the others.

39. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

    Find the probability of getting i) 3 heads ii) 2 heads and 1 tail iii) at least 2 heads.

40. 1500 families with 2 children were chosen at random in the state of Rajasthan and the following data were recorded:

Number of girls in the family	0	1	2
Number of families	211	814	475

If a family is chosen at random, find the probability that it has i) no girl ii) 1 girl iii) 2 girls iv) at most 1 girl v) more girls than boys.